Physics ATAR - Year 12

Gravity and Motion
Test 2
2017

Name: SOLUTIONS	Mark: / 51 = %

Time Allowed: 50 Minutes

Notes to Students:

1. You must include **all** working to be awarded full marks for a question.
2. Marks will be deducted for incorrect or absent units and answers stated to an incorrect number of significant figures.
3. **No** graphics calculators are permitted – scientific calculators only.

Question 1 **(9 marks)**

A future astronaut explorer standing on the surface of Mars experiences a gravitational force of 2.80 x10^2 N towards the center of Mars. The radius of Mars is 3.39 x10^6 m and its mass is 6.39 x10^{23} kg.

(a) Calculate the mass of the astronaut.

(3 marks)

$$F_g = \frac{Gm_m m_a}{r^2} \quad (1) \qquad m_a = \frac{Fr^2}{Gm_m} = \frac{280(3.39\times10^6)^2}{(6.67\times10^{-11})(6.39\times10^{23})} \quad (1)$$

$$= 75.5 \text{ kg} \quad (1)$$

(b) Calculate the gravitational field strength at the surface of Mars.

(2 marks)

$$\frac{F_g}{m} = a_g = \frac{Gm_m}{r^2} \quad (½) \qquad = \frac{(6.67\times10^{-11})(6.39\times10^{23})}{(3.39\times10^6)^2} \quad (½)$$

$$=3.71 \text{ ms}^{-2} \quad (\text{Nkg}^{-1}) \quad (1)$$

(c) The astronaut then moves away from the surface of Mars to an altitude where the gravitational force is 50.0 N. Calculate how far from the centre of Mars the astronaut is in terms of the radius of Mars (R_M)

(4 marks)

$$\frac{F_{50}}{F_r} = \frac{\left(\frac{Gm_m m_a}{r_{50}^2}\right)}{\left(\frac{Gm_m m_a}{r_m^2}\right)} \quad (½) = \frac{\left(\frac{1}{r_{50}^2}\right)}{\left(\frac{1}{r_m^2}\right)} \quad (½) = \frac{r_m^2}{r_{50}^2} = \frac{50}{280} \quad (1)$$

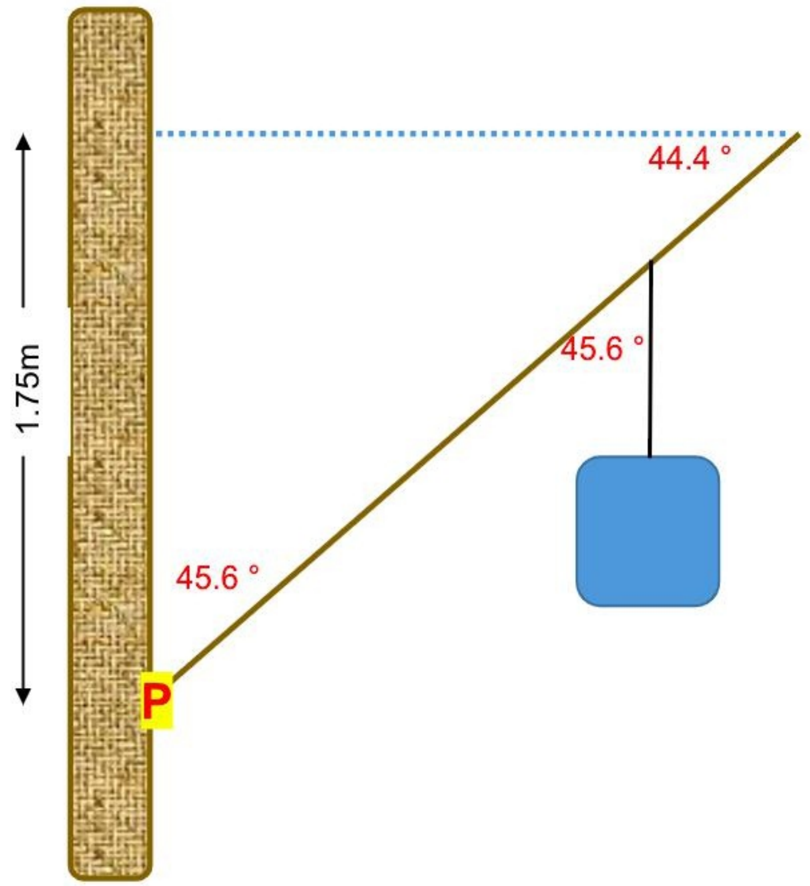
$$\frac{r_m}{r_{50}} = \sqrt{\frac{50}{280}} = 0.423 \quad (1)$$

$$r_{50} = 2.37\ r_m \quad (1)$$

Allow students to calculate r for F_{50} (8.02 x 10^6m), convert to multiple of r_M and then express ratio

Question 2 **(13 marks)**

A 2.50 m long beam of uniform mass of 55.0 kg is bolted to a vertical wall of a shop. A horizontal cable, attached at the end of the beam is attached at a point 1.75m above the bolt. A 15.0 kg shop sign is hung 2.00 m from the bolt.



a. Calculate the magnitude of the tension in the cable.

(4 marks)

½ ½

$\Sigma\tau = 0$ $\tau = rF\sin\theta$ $\theta = \cos^{-1}\left(\frac{1.75}{2.50}\right)$

$cwm = acwm$ = 45.6 ° ½ (and for determining 44.4°)

½ ½ ½

1.25(55x9.8)(sin45.6) + 2.00(15x9.8)(sin45.6) = 2.5(T)(sin44.4)

(or use R⊥=1.75)

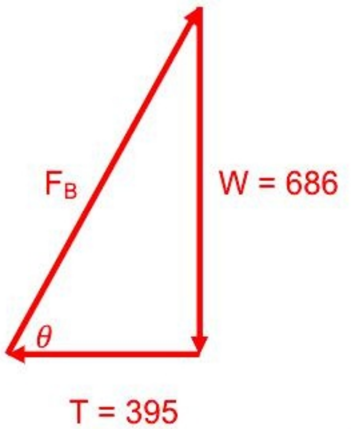
T = $\frac{691}{2.50\sin(44.4)}$

T = 395 N 1

b. Calculate the force of the bolt on the beam.

(4 marks)

Since system is in equilibrium, F_B is the equilibrant of T + ΣW (1)
Or some mention of system in Equilibrium ΣF = 0



$$|F_B| = \sqrt{395^2 + 686^2} = 792N$$ (1)

$$\theta = tan^{-1}\left(\frac{686}{395}\right) = 60.1°$$ (1)

<u>F_B = 792 N @ 60.1° above the horizontal</u> (1)

c. The shop keeper wishes to increase the visibility of the sign by mounting at the end of the beam. State the effect of this movement on the tension in the cable, including an explanation.

(5 marks)

- Increase
- Moving the sign to the end will increase the perpendicular distance to the pivot but the magnitude of the force does not change
- As τ =rFsinθ this will increase the counter clockwise torque.
- To keep the system in equilibrium, τcm = τacw
- Since the perpendicular distance to the cable remains constant, the Tension must increase

Question 3 **(17 marks)**

A satellite is said to be in a geosynchronous orbit if it's period of revolution is the same as the rotation of the Earth.

(a) Show the derivation of Kepler's 3rd Law

(3 marks)

$F_c = F_g$ (½)

$$\frac{mv^2}{r} = \frac{Gm1m2}{r^2}$$

$$v = \frac{2\pi r}{T}$$ (½)

$$\frac{m\frac{4\pi r^2}{T^2}}{r} = \frac{Gm1m2}{r^2}$$ (1 mark for logic)

$$\frac{4m2\pi r}{T^2} = \frac{Gm2}{r^2}$$

$$\rightarrow \quad \frac{r^3}{T^2} = \frac{Gm2}{4\pi^2}$$ (1)

(b) Calculate the altitude of the geosynchronous satellite above the earths surface.

(4 marks)

Set T = 24 hours x 60 x 60

= 86,400 s (1)

$$r_o = \sqrt[3]{\frac{GMT^2}{4\pi^2}} = \sqrt[3]{\frac{(6.67\times10^{-11})(5.97\times10^{24})(86400)^2}{4\pi^2}}$$ (½) (½)

$= 4.22 \times 10^7 - 6.38 \times 10^6$ (1)

$= 3.58 \times 10^7$ m (1)

(c) Calculate the orbital speed of the geosynchronous satellite.

(3 marks)

$v = \frac{2\pi R}{T}$ (1) or $v = \sqrt{\frac{GM}{r}}$ (1)

$= \frac{2\pi(4.22\times10^{7})}{86400}$ (1) $= \sqrt{\frac{(6.67\times10^{-11})(5.97\times10^{24})}{(4.22\times10^{7})}}$ (1)

$= 3.07\ \text{x}10^{3}\ \text{ms}^{-1}$ (1) $= 3.07\ \text{x}10^{3}\ \text{ms}^{-1}$ (1)

(d) State, making reference to an appropriate equation, by what factor the orbital speed of this satellite would change if the mass of the Earth was half of its accepted value, providing the radius remained constant.

(4 marks)

- from $\frac{r^3}{T^2} = \frac{Gm2}{4\pi^2}$, if r is constant, then $m \propto \frac{1}{T^2}$
- since v $v \propto \frac{1}{T}, then\ m \propto v^2$ and v $\propto \sqrt{m}$
- So if m was reduced by a factor of ½, then v would reduce by a factor of $\sqrt{1/2}$
- Reducing to 0.707 of its original speed.

OR

- From $v = \sqrt{\frac{GM}{r}}$, $v \propto \sqrt{M}$
- So if m was reduced by a factor of ½,
- then v would reduce by a factor of $\sqrt{1/2}$
- Reducing to 0.707 of its original speed.

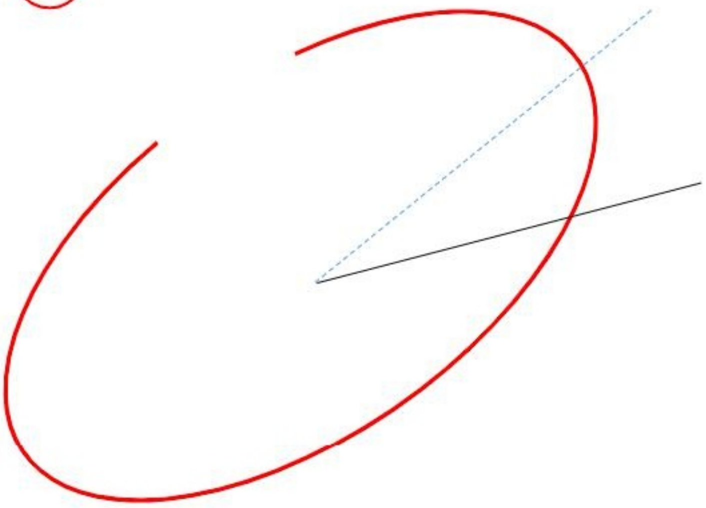
(e) Explain and state the difference between a geosynchronous orbit and a geostationary orbit, making use of the (not to scale) diagrams below.

(3 marks)

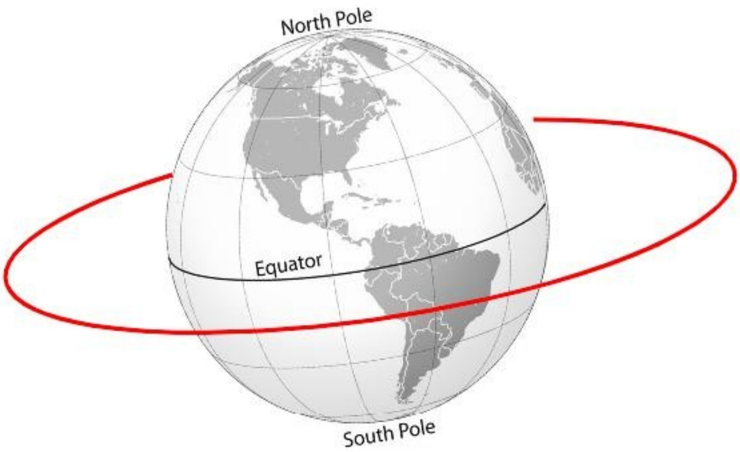
1 Geostationary → can only occur above the equator, satellite remains in fixed position relative to the surface of Earth.

1 Geosynchronous → can occur at any plane of rotation with respect to the equator, satellite will wobble / rise and fall in degrees of latitude relative to the surface of Earth.

1 1 mark for diagram



Geosynchronous

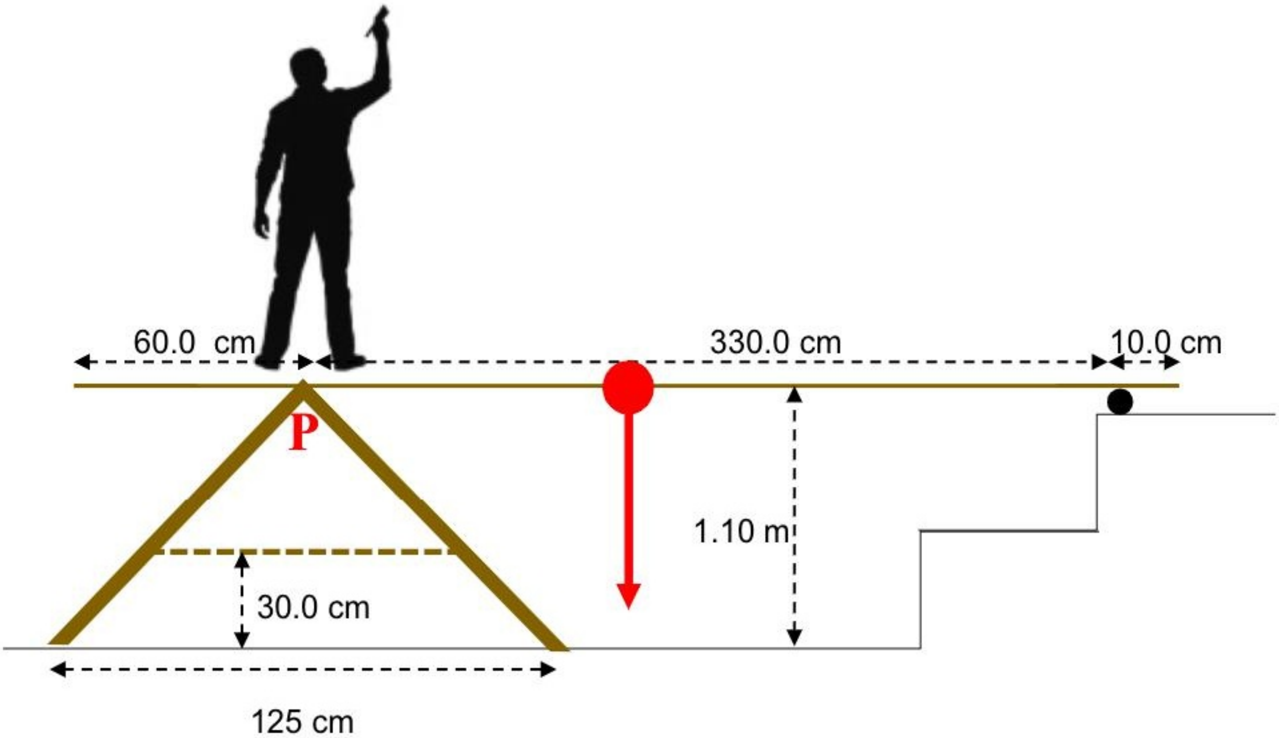


Geostationary

Question 4 **(12 marks)**

A painter of mass 80.0 kg is painting a ceiling above some stairs. He stands on uniform horizontal plank of mass 30.0kg supported at one end by the edge of the stairs and at the other by a trestle. The trestle stands on polished boards across which it can slide without significant friction.

A trestle is an 'A" shaped support (like a step ladder) with two sides hinged at the top. To prevent the sides of the trestle from splaying (spreading out) when the trestle is loaded, the two sides are joined by a length of rope.



(a) If the center of mass of the painter is directly above the center of the trestle, calculate the force exerted by the edge of the step on the plank.

(3 marks)

½ ½

$\Sigma\tau = 0$ $\tau = rF\sin\theta$

$cwm = acwm$

½ ½

$1.40 \times (30 \times 9.8) = 3.3\ F_R$

$F_R = \frac{1.4(294)}{3.3} = 125$ N upwards 1

(b) Calculate the minimum distance from the far left hand end of the plank that the painter could stand without the plank tipping.

(4 marks)

(½) (½)

$\Sigma\tau = 0$ $\tau = rF\sin\theta$

Tipping occurs when $F_R = 0$

$cwm = acwm$

(½) (½)

$(80\text{x}9.8) \text{ x } r_1 = 1.4\ (30\text{x}9.8)$

$r1 = \dfrac{1.4 \times (30\times9.8)}{(80\times9.8)} = 0.525\text{ m}$ (1)

$x = 0.600 – 0.525$

$= 0.0750\text{ m}$ (1)

(c) The painter moves to a point on the the plank which causes the force exerted by the edge of the step on the plank to be 500 N upwards. Calculate the tension in the rope of the trestle.

(5 marks)

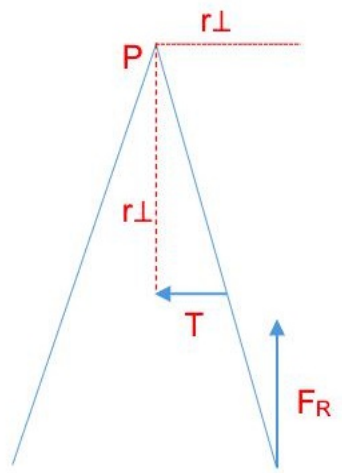
$\Sigma Fy = 0$

$= W_{painter} + W_{plank} + F_{trestle} + F_R$ (1)

$F_{trestle} = W + W - Fr$

$= (80\text{x}9.8) + (30\text{x}9.8) - 457$

$= 578$ N upwards (1)



$r\perp$ (for F_R) = 1.25 / 2 (½)
= 0.625 m

$r\perp$ (for T) = 1.10 m – 0.3 m = 0.8 m (½)

F_R at each leg is 578/2 = 289 (½)

$\Sigma\tau = 0$ $\tau = r\perp F$

$cwm = acwm$

0.8 T = 0.625 (289) (½)

T = 226 N (1)